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Poisson's Ratio for Rigid Plastic Foams*

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Synopsis

Poisson's ratio for several low-density plastic foams has been determined in both tension and compression. For polystyrene bead foams and a polyurethane foam, Poisson's ratio is greater in tension than compression. In compression, Poisson's ratio is not linear, showing a larger value below the yield strain and a value near zero for high strains. For 0.05 and 0.10 g/cc polystyrene bead foam, Poisson's ratios are $^1/_3$ in tension and $^1/_4$ in compression below the yield strain; at higher strains, the value in compression is in the range 0.03–0.07.

INTRODUCTION

Many new materials are now being used in load-bearing structures, and these materials may not be homogeneous, isotropic, or elastic solids. Two very different materials that are good examples are solid rocket propellants and rigid plastic foams. Their use as structural materials in complex geometries requires either a good stress analysis or large safety factors. For elastic stress analysis, any two of several equivalent elastic constants are required; modulus and Poisson's ratio are commonly used. If the material is not elastic, and most organic materials are not, pseudoelastic constants can be determined by making measurements at long times, essentially independent of time. Other measurements at shorter times and at different temperatures will yield the time–temperature dependence of the pseudoelastic constants. This report covers the determination of Poisson's ratio in tension and compression for several low-density rigid plastic foams under conditions yielding pseudoelastic values.

Poisson's ratio is defined for isotropic linear elastic solids in simple tension by the equation

$$\nu = -\epsilon_{\text{lateral}}/\epsilon_{\text{axial}} \tag{1}$$

where ϵ is strain.

Many elastic materials are not homogeneous or isotropic on a microscopic scale, and yet, Poisson's ratio is well defined by eq. (1). For rigid materials, $\nu \approx {}^1/_3$, and for rubbery materials, $\nu \approx {}^1/_2$. In general, multi-

* The term "Poisson's ratio" is used throughout this paper for brevity and because of common usage with nonelastic materials; however, it is recognized that a more appropriate term would be the "apparent Poisson's ratio" or "Poisson's effect."

phase materials will yield well-defined Poisson's ratio values if the constituents are uniformly dispersed. Examples of such composite materials on which Poisson's ratios have been reported in the literature are generally viscoelastic materials. In these cases, the lateral strain measurements are made at long times after extension to obtain results that are time independent. Smith[1] reports Poisson's ratio for small glass bead-filled 0–59 vol-% polyvinyl chloride to be 0.50 for small strains. Dilational measurements made by Fishman and Rinde[2] and Farris[3] on solid rocket propellants containing about 66 vol-% solids show Poisson's ratio to be near 0.50 at strains

TABLE I
Poisson's Ratio of Various Materials

Material	Strain region	Poisson's ratio	Reference
Polyurethane rubber	large	0.5	1
Polystyrene	small	0.336	10
Polystyrene (p = 15,000 psi)		0.353	
Filled polyvinyl chloride	small	0.5	1
0–59 vols % Glass beads	large	0.2–0.5	
Solid rocket propellants	<5%	0.5	2,3
66 vol-% Fillers	>5%	~0.25	
Polypropylene (−50 to 40°C)	small	0.33–0.35	9
Polypropylene (80°C)	small	0.45	
Flexible polyurethane foam, 47% voids	0–140%	$^1/_4$	4
Natural rubber foam, 50–90% voids	0–10%	0.33	5
Polystyrene foam compression	10–55%	0.03	6
Carbon foam			
0.05 g/cc Parallel	small	0.33	7
0.05 g/cc Perpendicular	small	0.12	
0.10 g/cc Parallel	small	0.24	
0.10 g/cc Perpendicular	small	0.15	

below 5%. Their data also show the time dependence of lateral straining at several strain rates. Blatz and Ko[4] have determined Poisson's ratio in simple tension and in strip biaxial and homogeneous biaxial tension on a flexible polyurethane foam rubber containing 47 vol-% voids and found a value of $^1/_4$. Gent and Thomas[5] found Poisson's ratio to be $^1/_3$ for flexible foam rubbers and independent of void content in the range of 50–90% voids. Shaw and Sata[6] found the Poisson's ratio of some undefined polystyrene foam to be 0.03 in compression at high strains. Rand[7] has reported Poisson's ratios of anisotropic carbon foams to be in the range of 0.12–0.15 in the perpendicular-to-rise direction and 0.25–0.33 in the parallel-to-rise direction.

Freudenthal and Henry[8] have investigated the variability of Poisson's ratio for linear viscoelastic materials. Their analysis shows Poisson's ratio to be dependent upon time and loading conditions (such as constant strain, stress). For a Kelvin body, Poisson's ratio increases with time and approaches $^1/_3$ at long times. For a Maxwell body, Poisson's ratio is equal to $^1/_3$ at $t = 0$ and goes to 0.50 as time increases. In both of these cases the Poisson's ratios are dependent on the assumed ratio of $G/K = {}^3/_8$.

Rigbi[9] presents the temperature dependence of ν for several plastics. With Waterman's dynamic data on isotactic polypropylene, Poisson's ratio is shown to be nearly constant at 0.33–0.35 for the temperature range $-50°$ to 40°C, and then rises to about 0.45 at 80°C.

Hughes et al.[10] have measured the pressure dependence of Poisson's ratio of solid polystyrene. They found $\nu = 0.336$ at $P = 0$ and $\nu = 0.353$ at $P = 15{,}000$ psi at 31°C. The above review of Poisson's ratio of various materials is summarized in Table I.

Ko[11] has used the geometric shapes of the interstices of two types of close packing of uniform spheres, hexagonal closest packing (*hcp*) and face-centered cubic (*fcc*) closest packing, to represent the structure of actual foams. Equivalent elastic constants for these structures are calculated in terms of the slenderness of a thread l^2/A, where l is the length of a thread (strut) and A is its cross-sectional area, and of the elastic constants of the base polymer. His calculations showed a model containing $^2/_3$ of the interstices of *hcp* and $^1/_3$ of those of *fcc* packing correlated with his data of foam rubbers.

DEFINITION AND DETERMINATION OF VARIOUS POISSON'S RATIOS

Rigid plastic foams in compression have been reported to have Poisson's ratios very near zero. From the experimental point of view, this means that for any given principal strain there will be little or no lateral strain. If Poisson's ratio in compression is near zero, then the problem of measuring these small lateral strains directly and accurately becomes very difficult, although a limiting value may be readily determined down to the limit of measurement. A much more simple and accurate method for determining Poisson's ratio on rigid plastic foams is to use the buoyancy technique.[12,13] This method measures a bulk property volume change, which is directly related to the strains. It has the experimental advantage that as ν goes to zero, the volume change increases. For polystyrene bead foams which are not homogeneous on a small scale, it has the additional advantage in that it measures an average property of the foam.

Equations

The necessary equation relating volume change and Poisson's ratio is derived from the following conditions:

$$v/v_0 = \lambda_1\lambda_2\lambda_3 \tag{2}$$

$$\lambda = 1 + \epsilon \tag{3}$$

$$\nu = -\epsilon_2/\epsilon_1 \tag{4}$$

$$\lambda_2 = \lambda_3 \tag{5}$$

where v_0 is the initial volume, v is the volume at some strain ϵ, λ is the extension ratio l/l_0, where l is a length, and ν is Poisson's ratio. Equations (3) to (5) are substituted into eq. (2); and after differentiation, retaining only linear terms of ϵ and rearranging, the required equation is obtained:

$$\nu = \frac{1}{2}\left[1 - \frac{1}{v_0}\left(\frac{\partial v}{\partial \epsilon}\right)\right]. \tag{6}$$

Poisson's ratio is calculated using eq. (6) and the slope of the straight line obtained from a plot of volume versus axial strain. Equation (6) yields the correct value of Poisson's ratio, $^1/_2$, for an incompressible material, i.e., no volume change.

For the special case of Poisson's ratio being equal to zero, that is, no lateral strain ($\lambda_2\lambda_3 = 1.0$), the volume change is a function of only λ_1 [eq. (2)]. In this case, λ_2 is probably equal to λ_3 even for anisotropic foams.

For the general case of anisotropic foams, λ_2 will equal some function of λ_3, and for most foams the relationship should be of the form $\epsilon_2 = b\epsilon_3$, where b is a constant. Equation (6) can be generalized for anisotropic materials, using this relationship with the result

$$\nu_{\lambda_2} = \frac{1}{1 + (1/b)}\left[1 - \frac{1}{v_0}\left(\frac{\partial v}{\partial \epsilon}\right)\right] \tag{7}$$

where b must be obtained independently.

In the literature,[1-4] Poisson's ratio measurements have been published for very extensible materials at strains up to several hundred per cent. For these materials, a new definition of Poisson's ratio was required to obtain a linear relationship between strain and volume change at high strains. The relationship used by Smith[1] and Blatz and Ko[4] that correlates the experimental data well is based on the logarithmic or Hencky measure of strain, Poisson's ratio being defined as

$$\nu_{\log} = -\frac{\ln \lambda_{\text{lateral}}}{\ln \lambda_{\text{axial}}}. \tag{8}$$

For small strains, eq. (8) may be linearized to eq. (1), showing that this logarithmic Poisson's ratio has the same significance of Poisson's ratio in infinitesimal elasticity. Since much of the literature work was done dilatometrically, Poisson's ratio is obtained from plots of log v/v_0 versus log λ, where the slope of the straight line is equal to $1 - 2\nu_{\log}$. Since $\nu_{\log}$ reduces to ν at small strains, these values for Poisson's ratio are directly comparable at small strains.

EXPERIMENTAL

The buoyancy technique used for this work has been used previously to measure Poisson's ratio on various materials.[13] Details of the sample-holding fixture vary, depending upon the material properties and geometry; however, the basic features are the same. Our sample holder consisted of two parallel plates of $^1/_4$-in.-thick steel connected by threaded bolts. The foam sample was bonded to both of these plates and extended or compressed by turning nuts which forced the parallel plates to move. Test specimens were generally 1 in. × 1 in. × 3 in. long, but in all cases at least three times as long as the width or thickness directions. Tests were made on samples uncoated or coated with a thin layer of an RTV silicone rubber to prevent water penetration. Duplicate tests showed no differences due to coating.

Each determination consisted of weighing and measuring the foam specimens to calculate their initial volume before they were bonded into the weighing fixture and coated with the RTV rubber. After the silicone rubber had cured, the sample and fixture were submerged in water for at least a half-hour before the first weight was recorded. The weights were obtained by suspending the fixture on a fine wire from an analytical balance arm and totally submerging it in water. The amount of axial extensions was measured directly using a micrometer at two places on the outside of the steel plates, and the average value was used to calculate the strain. Straining was continued until a crack was observed in tension or to 5% strain in compression.

Two direct methods of measuring lateral strains were used. One method was to stamp with ink a square grid pattern on foam specimens of 1, 2, and 3 in. in length and 1 in. × 1 in. in cross section. These specimens were tested in an Instron test machine at a low crosshead speed, and a photograph was taken about every 0.1% strain. The axial and lateral displacements were read from the films on a film reader using a magnification of about 5×. The precision on measuring the lateral displacements limited the accuracy of Poisson's ratio values obtained by this method. The other method was to extend the specimens a known amount and measure the latter displacement with a disk micrometer. This method was not very accurate at strains below 5–10%.

RESULTS

Poisson's ratio values have been determined on both rigid and flexible foams in tension and rigid foams in compression. Three different types of foams are represented; isotropic, orthotropic, and anisotropic. The PSB and cellular silicone are isotropic; polyurethanes and polypropylene foams are orthotropic; and the extruded polystyrene and polyethylene foams are anisotropic. In general, the isotropic foamed materials show tensile Poisson's ratio values equivalent to solid materials while Poisson's ratio of the orthotropic and anisotropic foams are governed by the foam direction and structure. Poisson's ratio values in compression are less than the tensile values, showing that cellular buckling probably occurs to some extent before the yield strain is reached; after the yield strain, buckling becomes pronounced and Poisson's ratio approaches zero. Poisson's ratio in compression at strains higher than the yield strain are density dependent, increasing from about 0.03 at 0.05 g/cc to about 0.15 at 0.2 g/cc density.

Polystyrene Bead Foam

PSB foams of 0.05 and 0.10 g/cc are used as structural materials, and they are both rigid and isotropic. Our foams were molded in the form of large blocks from Dow SD-505 expandable polystyrene beads. Average stress–strain curves are shown in Figure 1.

The experimentally determined values for Poisson's ratio in tension are presented in Table II and the compression values, in Table III. Values

TABLE II
Poisson's Ratio and Modulus of Various Foams in Tension

Foam	Density, g/cc	Direction	Poisson's ratio: Average value	Poisson's ratio: Range	Modulus, psi
Polystyrene bead	0.05	isotropic	0.32	0.30–0.36	5,500
Polystyrene bead	0.10	isotropic	0.32	0.28–0.34	11,000
Cellular silicone rubber UCC Y-3260	0.51	isotropic	0.33	0.30–0.34	55
Rigid polyurethane polyether type	0.10	foam rise	0.40		8,500
Rigid polyurethane polyether type	0.20	foam rise	0.31		16,000
Polypropylene	0.07	foam rise	0.25	0.24–0.25	4,000
Extruded polystyrene	0.065	foam rise	$\nu_E = 0.46$	$\nu_W = 0.23$	13,000
		width	$\nu_E = 0.53$	$\nu_{FR} = 0.26$	
		extrusion	$\nu_W = 0.40$	$\nu_{FR} = 0.20$	
Extruded polystyrene	0.10	foam rise	$\nu_E = 0.47$	$\nu_W = 0.23$	
Polyethylene	0.04	foam rise	$\nu_E = 0.33$	$\nu_W = 0.66$	200

shown are average values obtained from four or five determinations and the range of values obtained. Also shown are modulus values obtained at low testing rates in accordance with testing procedures outlined in ASTM D-1623 using Type B specimens.

Poisson's ratio values in tension on both densities of PSB foam were found to be the same, and also approximately the same as full-density polystyrene. No differences were found due to anisotropy when determinations were made in each of two perpendicular directions on 0.05-g/cc density foam in tension. Direct measurements confirmed the above results within the limits of accuracy of the methods used.

TABLE III
Poisson's Ratio and Modulus of Various Foams in Compression

Foam	Density, g/cc	Direction	Below yield strain		Above yield strain	
			Average value	Range	Average value	Modulus, psi
Polystyrene bead	0.05	isotropic	0.22	0.18–0.27	0.03	4,000
Polystyrene bead	0.10	isotropic	0.25	0.22–0.27	0.07	7,000
Rigid polyurethane polyether type	0.10	foam rise	0.20		0.09	9,000
Rigid polyurethane polyether type	0.20	foam rise	0.29		0.14	20,000

Figure 2 shows representative curves of our volume–strain data for the several foams in tension. In this figure, and several others, a factor A has been added to the volume of some of the curves so that data from specimens of different initial volumes can be plotted on the same scale. The curves for the two PSB foams are seen to be linear up to the failure strain of about 2–3%.

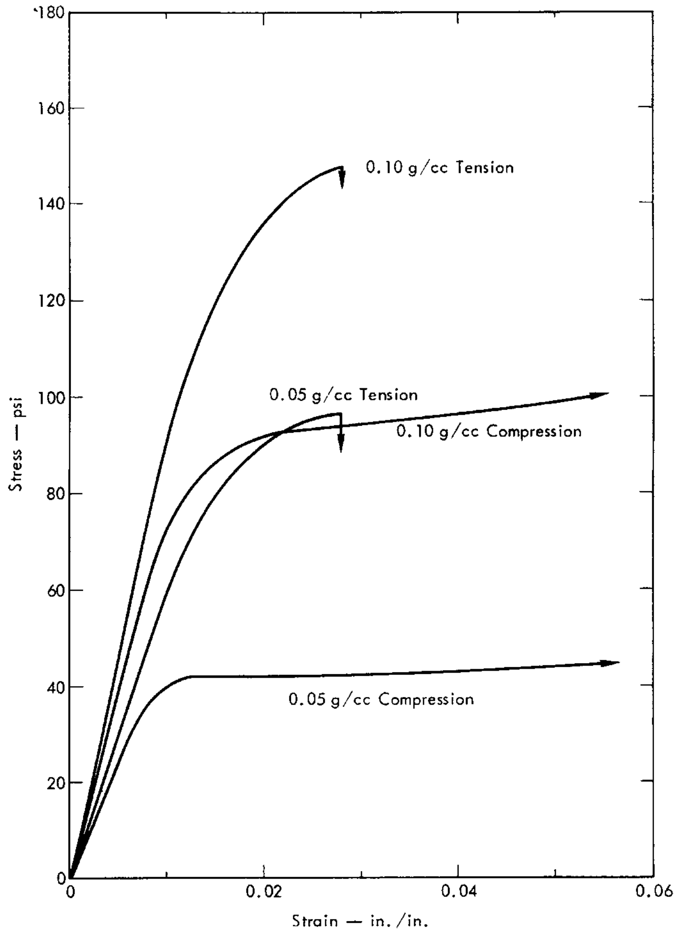

Fig. 1. Stress–strain curves for polystyrene bead foams.

In compression, rigid plastic foams show two distinct regions of mechanical behavior, nearly elastic behavior below the yield strain and crushing with little stress increase at higher strains. These regions are reflected in the Poisson's ratio values as a large value is found at low strains and a value near zero is found at high strains. For the PSB foams, $\nu = {}^1/_4$ below about 1% strain and is equal to 0.03 to 0.07 at higher strains. These Poisson's ratio values at both high and low strains seem to be density dependent, as

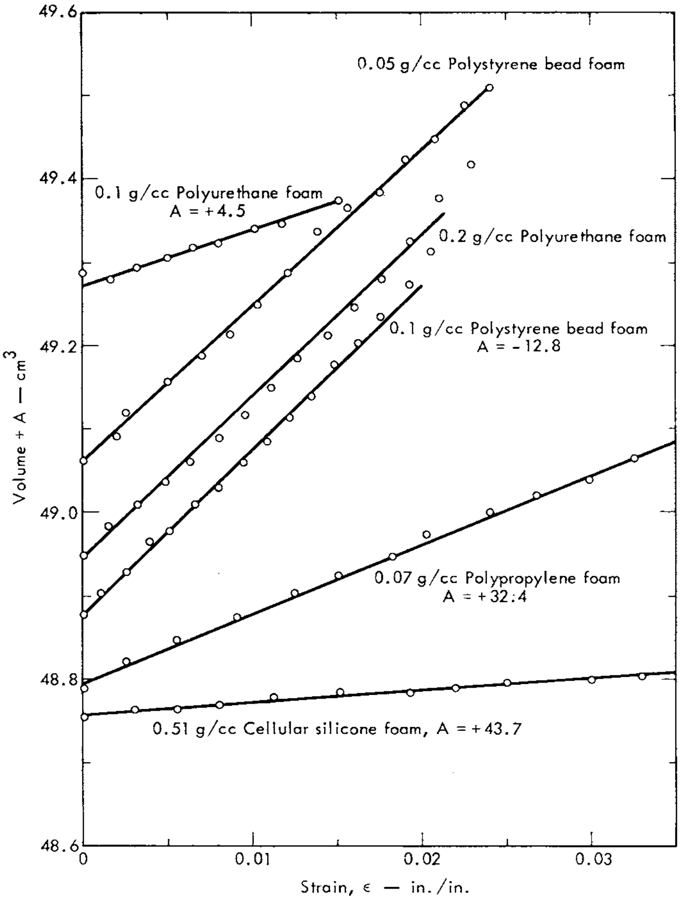

Fig. 2. Volume–strain curves for various foams in tension. A factor *A* has been added to the volume of some of these curves to physically separate them.

shown by the PSB and polyurethane foam data in Table III. For these foams, Poisson's ratio is greater in tension than in compression.

Figure 3 shows representative volume–strain curves for the PSB and polyurethane foams in compression. This figure shows the volume–strain curves to be linear in the region below 1–2% strain, showing a break and becoming linear again but with a higher slope. The curves in Figure 3 continue to 5% strain, the limit of our data, but the points are not shown. The break in the volume–strain curves at about 1–2% strain corresponds to the yield point on the stress–strain curves, Figures 1 and 4, and therefore is not surprising. The yield point indicates the transition from nearly elastic behavior at small strains to crushing of the foam at high strains. During crush little or no lateral straining occurs, and therefore Poisson's ratio is expected to be near zero. Shaw and Sata's value[6] of 0.03 would appear to correspond to a foam that is being crushed. Reference to their paper shows

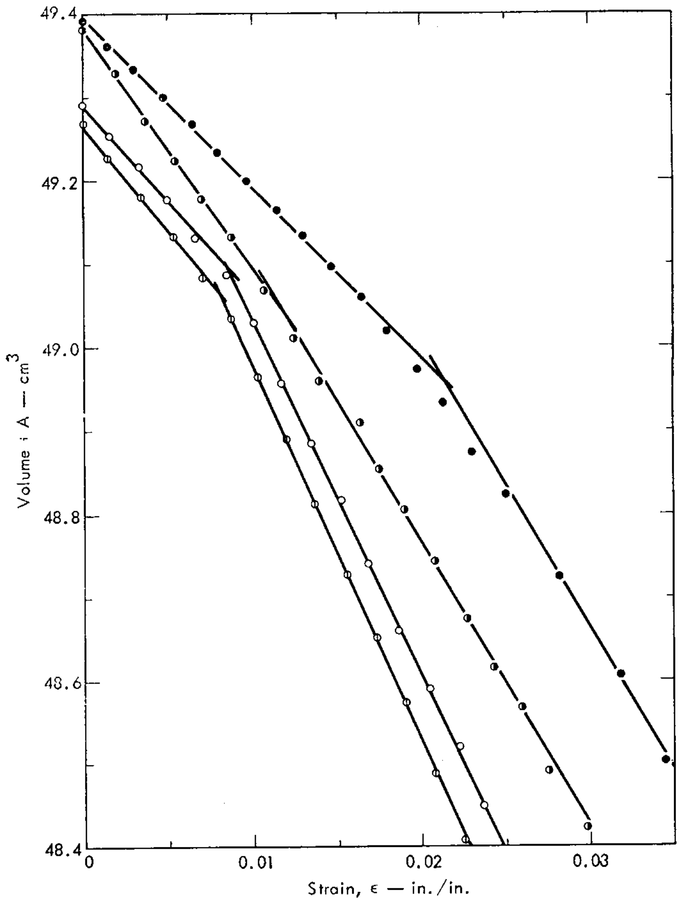

Fig. 3. Volume-strain curves for PSB and polyurethane foams in compression. A factor A has been added to the volume of some of these curves to physically separate them: (●) 0.2 g/cc Pu, $A = 0$; (◑) 0.1 g/cc Pu, $A = +0.5$); (O) 0.1 g/cc PSB, $A = 0$; (⌽) 0.05 g/cc PSB, $A = +0.8$.

that their first lateral strain point is at about 12% strain, well beyond the yield strain, and therefore does represent Poisson's ratio during crushing.

It could be expected that different values for Poisson's ratio would be found in tension above and below the yield strain. For the PSB foams we tested, failure occurs at or before the yield strain, and therefore only a single Poisson's ratio is found. However, the extruded polystyrene foams (see Fig. 5) do show yield points in tension and lateral collapse of the foam structure, with a corresponding increase in Poisson's ratio.

Cellular Silicone

The cellular silicone foam Y-3260 is made by incorporating a temporary filler in the base polymer, molding and curing into sheets, and leaching out the filler. The foam is of 0.51-g/cc density and is flexible and isotropic.

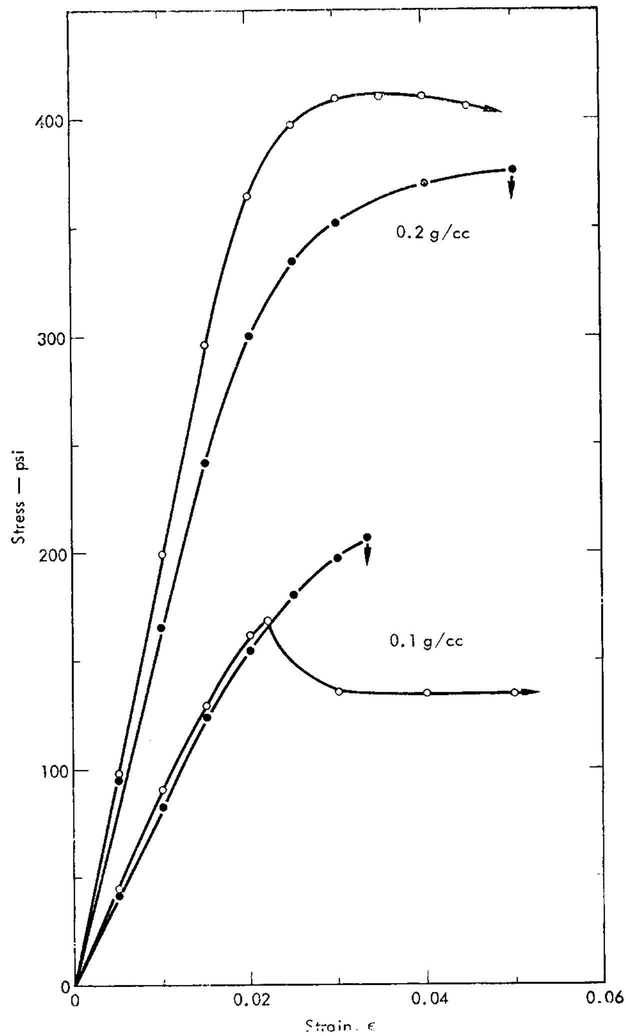

Fig. 4. Stress-strain curves for polyurethane foams: (●) tension; (O) compression.

It is, therefore, very similar to foams used by Blatz and Ko and by Gent and Thomas to obtain Poisson's ratio values by direct methods. Blatz and Ko found $\nu = {}^1/_4$ for strains from about 5–104% on their polyurethane foam rubber, while Gent and Thomas found $\nu = {}^1/_3$ for strains from 0–10% on natural rubber foams independent of foam density. Our result of 0.33 obtained by the buoyancy technique at strains below 10% is, therefore, very satisfying.

Polyurethane Foams

The rigid urethane foams are made from polyethers and are orthotropic. Orthotropic behavior is two-dimensional; that is, the properties in the foam

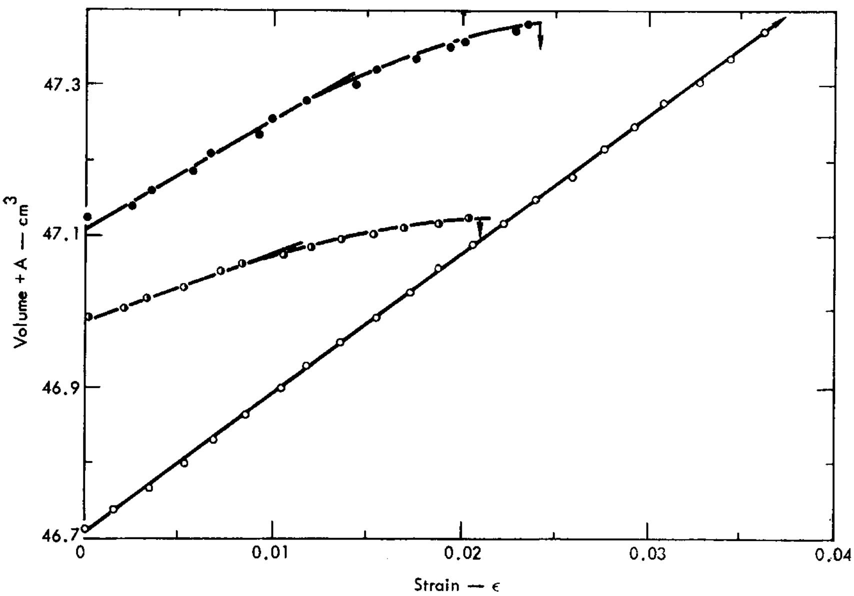

Fig. 5. Volume–strain curves for extruded polystyrene foam in tension. A factor A has been added to the volume of some of these curves to physically separate them: (●) $A = -1.0$, foam rise direction; (◑) $A = -0.6$, width directions; (○) $A = 0$, extrusion direction.

rise direction are different from the other two directions, which are approximately the same. In this case we determined Poisson's ratio in the foam rise direction, and since $\lambda_2 = \lambda_3$, eq. (6) can be used to determine ν in this direction. The degree of orthotropic behavior is shown by the strength ratios for the foam rise to lateral directions; for the 0.1-g/cc foam, the ratio is 3 to 1 and for the 0.2-g/cc foam, the ratio is 4 to 3. Average stress–strain curves in the foam rise direction are shown in Figure 4.

Poisson's ratio values for the polyurethane foams listed in Tables II and III are the results of single tests. At the 0.1-g/cc density, a high ν value would be expected in tension and a correspondingly lower value in compression due to the high degree of orthotropic behavior. At the 0.2-g/cc density, ν values comparable to the PSB foams would be expected since this foam is nearly isotropic. By comparing the 0.2-g/cc polyurethane foam with the PSB foams, it is seen that the tensile ν values are essentially the same and independent of density, while the compression values are density dependent.

Polypropylene Foam

The polypropylene foam used was made from a modified polypropylene. It is semirigid and orthotropic. The orthotropic behavior is two-dimensional, as are the polyurethane foams, and eq. (6) was used to calculate Poisson's ratio in the foam rise direction. For strains below 10%, Poisson's ratio in tension was found to be 0.25.

Extruded Polystyrene Foam

The extruded polystyrene foams are commercial foam and a special high-density foam. These extruded polystyrene foams are anisotropic in all three directions. Therefore, to obtain useful data on this foam, some knowledge of the lateral strains must be obtained to determine the ν value for the different directions. From direct measurements of the lateral deflections during tensile elongation, the following ratios of lateral straining were measured. When the principal straining is in the foam rise (FR) direction, $2\epsilon_W = \epsilon_E$; in the width (W) direction, $2\epsilon_{FR} = \epsilon_E$; in the extrusion (E) direction, $2\epsilon_{FR} = \epsilon_W$. More precision cannot be given the above numbers because our measurement technique was not accurate enough.

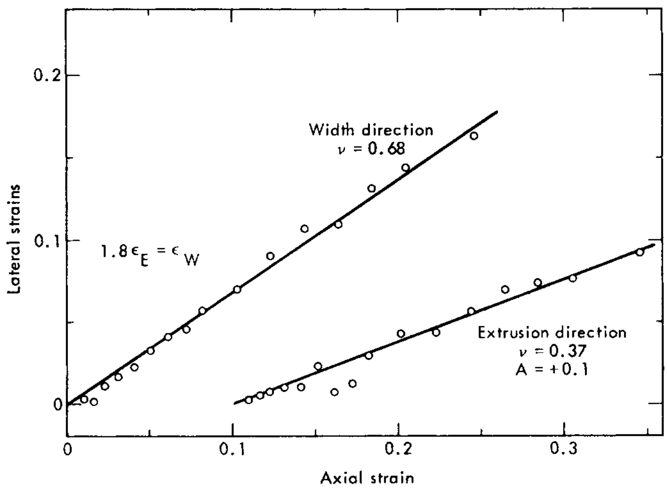

Fig. 6. Axial vs lateral strains for polyethylene foam in tension. A factor A has been added to the axial strain in the extrusion direction to physically separate the curves.

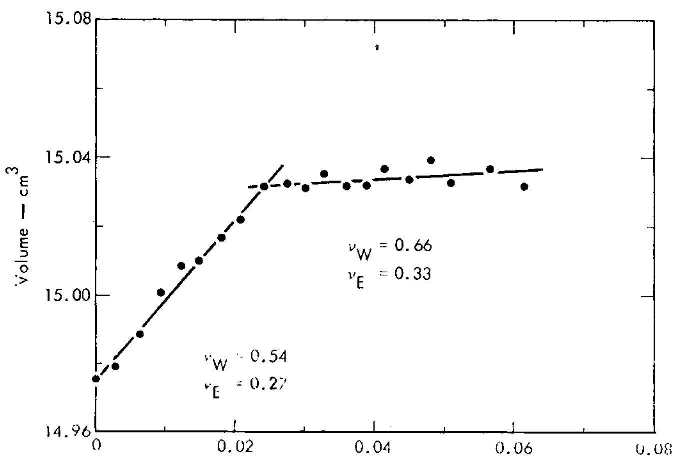

Fig. 7. Volume–strain for polyethylene foam in tension.

Results on extruded polystyrene foam are further complicated by the fact that in tension these foams show a yield strain before failure on the volume-versus-strain plots. Because of this yield region, Poisson's ratio is constant up to the volumetric yield strain (1.5–2% strain) and then increases with strain. Extruded polystyrene in the extrusion direction is an exception in that a completely linear volume–strain curve is obtained, while its stress–strain curve has the same general shape as the other two directions. The curves for extruded polystyrene foam in the three directions are shown in Figure 5. In all the determinations the curves are definitely linear before the yield point.

Polyethylene Foam

The polyethylene foam used was commercial extruded foam and is flexible and three-dimensionally anisotropic. When direct measurements of the lateral strains were made (Fig. 6), normal behavior was observed and ν_W = 0.68, ν_E = 0.37 were found when the foam was strained in the foam rise direction. However, when buoyancy determinations were made, they showed a yield point at about 2% strain. Using the b value from the above direct measurement, $1.8\ \epsilon_E = \epsilon_W$, in eq. (7), Poisson's ratio was found to be lower, ν_W = 0.54 and ν_E = 0.27 at strains below 2%, and then increases to approximately the same values were obtained by the direct method (see Fig. 7). Therefore, it is interesting to note that the use of the buoyancy technique, with its high degree of precision, allowed the obtaining of more information than was available from our direct measurements.

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